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VARIANCE IN SCINTILLATOR LIGHT COLLECTION

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Electronics Division

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ABSTRACT

A Monte-Carlo program for the "GEORGE" Computer at Argonne National Laboratory was used to compute the average path length and number of diffuse reflections of a light ray originating at a given point in a cylinder before it is absorbed in a "black" concentric circle in the bottom of the cylinder. The variance of these numbers is estimated and indicates light-trapping effects. Total internal reflection at the exit ("black") interface is included. The average number of reflections increases rapidly as the crystal length exceeds one diameter.

INTRODUCTION

Scintillation gamma-ray spectrometers have, like all spectrometers, a certain instrumental line width. This is due in part to the fact that a gamma photon of given energy will produce only a finite number of photoelectrons at the photocathode of the photomultiplier tube. For instance a 660-kev photon absorbed in NaI(Tl) will give about 2000 photoelectrons on the average. Since they are the result of a large number of statistically independent interactions at the photocathode, the number of photoelectrons resulting from any given gamma ray will deviate from the mean with an rms deviation of about the square root of the mean.

In practice, there are other effects that increase the instrumental line width even further, such as the variations in efficiency of light collection from different parts of the crystal. If, on the average, light from one part of the crystal has more diffuse reflections off the white reflector that covers the walls of a crystal than light from another part, then any absorption in the walls will affect one region more than the other. The relative variance (variance/mean^2) caused by this effect in the case of small absorption is $c^2\sigma^2$ where "c" is the absorption coefficient and σ^2 is the variance among the mean number of diffuse reflections of a light ray from different points in the crystal before it gets absorbed in the photocathode.[1] A similar formula also holds for the effects of absorption of

[1] Ernst Breitenberger, "Scintillation Spectrometer Statistics," Progress in Nuclear Physics, Vol. 4, Pergamon Press, New York, (1955) p. 56.

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light in the volume of the crystal. The authors believe that the absorption in the volume of the crystal is not important, however, because of the similar performance of crystals of widely different sizes but with the same proportions.

MONTE-CARLO CALCULATION

A Monte-Carlo program was computed on the "GEORGE" computer at Argonne National Laboratory to determine the average path length and number of diffuse reflections of a ray of light originating at given points in a cylinder before absorption in a dark concentric circle in the bottom of the cylinder. Figure 1 represents schematically a cylinder of unit diameter and height H. There is dark absorbing photocathode of diameter D on the bottom. A light ray originates at some selected point "p" in the interior and travels in a random direction. If the pulse intersects the top or side or the part of the bottom that is not part of the photocathode, a diffuse reflection occurs and the light travels in a new random direction, with the probability per solid angle of any particular direction being proportional to the cosine of the angle between that direction and the normal. Since the index of refraction of the crystal is usually different than that of the glass of the photocathode, there is a total internal reflection at the photocathode if the ray strikes it at an angle shallower than a certain critical angle. For NaI(Tl) and glass, this angle has a cosine of about 0.575. If the ray strikes the photocathode at an angle steeper than the critical angle, it is absorbed.

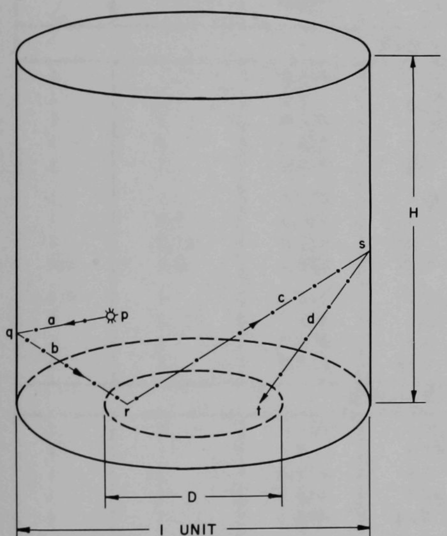

Fig. 1

In Figure 1, a ray originates at "p," has diffuse reflections at "q" and "s," a total internal mirror-like reflection at "r," and is absorbed at "t," giving a total of two diffuse reflections and a total path length of a + b + c + d.

Many histories from each point of origin were run. The average and the mean square deviation from the average were computed for each point. Table I gives the variance of these averages corrected for the statistics of the finite number of cases in the Monte-Carlo computation, and also gives the overall average for a number of crystal geometries.

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The figures presented are for the special, and in many cases unrealistic, case of a uniform distribution of points of origin throughout the crystal. Also, multiple Compton scatterings of a gamma photon would deposit energy in several regions of the crystal, thus partially averaging out the variations of light collection. The figures do illustrate, however, the general features of the problem of light collection.

Table I

PATH LENGTH AND NUMBER OF DIFFUSE REFLECTIONS

1 Height	2 Cathode Diameter	3 Cosine of Critical Angle	4 5 Overall Average	6 7 Variance of Means	8 Standard Deviation from One Point
			Path Length		
1	1	0.125	3.63 ± 0.038	0.34 ± 0.013	3.41
1	1	0.5	4.58 ± 0.059	0.12 ± 0.022	4.48
1	1	0.575	5.15 ± 0.046	0.087 ± 0.016	5.06
1	1	0.625	5.63 ± 0.061	0.026 ± 0.033	5.52
1	1	0.875	16.9 ± 0.37	1.1 ± 1.2	16.5
1	1	0.575	5.15 ± 0.046	0.087 ± 0.016	5.06
1	0.5	0.575	22.5 ± 0.43	0.46 ± 0.9	23.6
1	0.25	0.575	92.3 ± 1.5	-12.4 ± 13.0	92.5
0.5	0.5	0.575	11.5 ± 0.22	-0.15 ± 0.25	12.6
0.5	1	0.575	2.57 ± 0.024	-0.002 ± 0.0044	2.66
1	1	0.575	5.15 ± 0.046	0.087 ± 0.016	5.06
2	1	0.575	11.8 ± 0.065	3.10 ± 0.057	11.6
4	1	0.575	30.6 ± 0.52	70.3 ± 1.6	31.1
			Diffuse Reflections		
1	1	0.125	4.72 ± 0.055	0.76 ± 0.027	5.01
1	1	0.5	5.94 ± 0.070	0.30 ± 0.044	6.36
1	1	0.575	6.71 ± 0.065	0.33 ± 0.031	7.10
1	1	0.625	7.24 ± 0.27	0.10 ± 0.076	7.62
1	1	0.875	21.6 ± 0.48	1.9 ± 2.0	21.5
1	1	0.575	6.71 ± 0.065	0.33 ± 0.031	7.10
1	0.5	0.575	32.8 ± 0.64	0.86 ± 2.0	35.3
1	0.25	0.575	138 ± 2.3	-29 ± 30	139
0.5	0.5	0.575	21.9 ± 0.44	-0.6 ± 1.3	24.3
0.5	1	0.575	3.94 ± 0.039	0.034 ± 0.012	4.27
1	1	0.575	6.71 ± 0.065	0.33 ± 0.031	7.10
2	1	0.575	13.7 ± 0.080	4.86 ± 0.086	14.4
4	1	0.575	33.3 ± 0.64	88 ± 2	35.1

Notice in the table that, even though the presence of internal reflection (higher cosines of critical angle) increases the average number of reflections, the variance among the averages from different parts of the crystal is reduced and thus may actually be an improvement of the performance of a given crystal. Notice also how rapidly the variance increases as the crystal gets longer than 1 diameter. The average of

the rms deviations computed for each point of a given geometry is given in the last column. For the number of reflections, notice that it is more like the mean rather than the square root of the mean.

The authors wish to thank the members of the Applied Mathematics Division of Argonne National Laboratory, and in particular, Loretta Kassel for many helpful discussions and for hours spent in tutoring one author in the programming of Monte-Carlo problems. Thanks are also due to T. Brill, former Director, Electronics Division, for encouragement and support of this work, and to S. I. Baker and I. S. Sherman of the Electronics Division for their work on the appendix.

APPENDIX

The overall average, which is listed in column four of Table I, is given by

$$\overline{C} = \sum_{i=1}^{N} \overline{C}_i \frac{1}{N} \quad ,$$

where $\overline{C}_i$ is the average number of reflections of rays leaving the i^{th} point in the crystal, and N is the number of points for which case histories were run.

In column five the estimated error in $\overline{C}$ due to taking a finite number of case histories:

$$\sigma^2_{\overline{C}} = \frac{1}{N^2} \sum_{i=1}^{N} \sigma^2_i \quad ,$$

is listed. The Monte-Carlo variance at the i^{th} point is

$$\sigma^2_i = \frac{1}{M(M-1)} \sum_{k=1}^{M} (C_{ki} - \overline{C}_i)^2 \quad ,$$

where C_{ki} is the number of reflections for one case history at the i^{th} point, and M is the number of case histories at one point.

The expressions

$$\langle C^2_i \rangle = \frac{1}{M} \sum_{k=1}^{M} C^2_{ki} \quad \text{and} \quad \langle C_i \rangle^2 = \left(\sum_{k=1}^{M} C_{ki} \right)^2$$

were calculated by the computer. The difference between the two, $\langle C^2_i \rangle - \langle C_i \rangle^2$, is equal to

$$\frac{1}{M} \sum_{k=1}^{M} (C_{ki} - \overline{C}_i)^2 \quad ,$$

shown above.

In column six is found the corrected variance of means (variance of means minus the average Monte-Carlo variance at one point) given by

$$\left[\frac{1}{N}\sum_{i=1}^{N}(\overline{C}_i - \overline{C})^2\right] - V$$

where

$$\overline{C} = \frac{1}{N}\sum_{i=1}^{N}\overline{C}_i \quad ; \quad V = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^2 \quad .$$

Column seven gives the confidence limits (5% and 95%) placed on the value of the corrected variance of means. The following expression results after simplification:

$$\pm\frac{V}{N}(\chi_5^2 - N) \mp \frac{V}{N}(N - \chi_{95}^2) \cong \pm\frac{V}{2N}(\chi_5^2 - \chi_{95}^2)$$

where χ_5^2 is the 5% value and χ_{95}^2 is the 95% value of $\chi^2\ \{N - 1\}$.

Column eight contains values of the average standard deviation over the crystal.

$$\sigma_C = \sqrt{\frac{1}{N}\sum_{i=1}^{N} M\sigma_i^2}$$